Optimizing Inspection Operations

Renee Clarke & Darlena Kern

Mentors:

Dr. Elsayed, Dept. of ISE, Rutgers University
Dr. Xie, Dept. of Statistics, Rutgers University

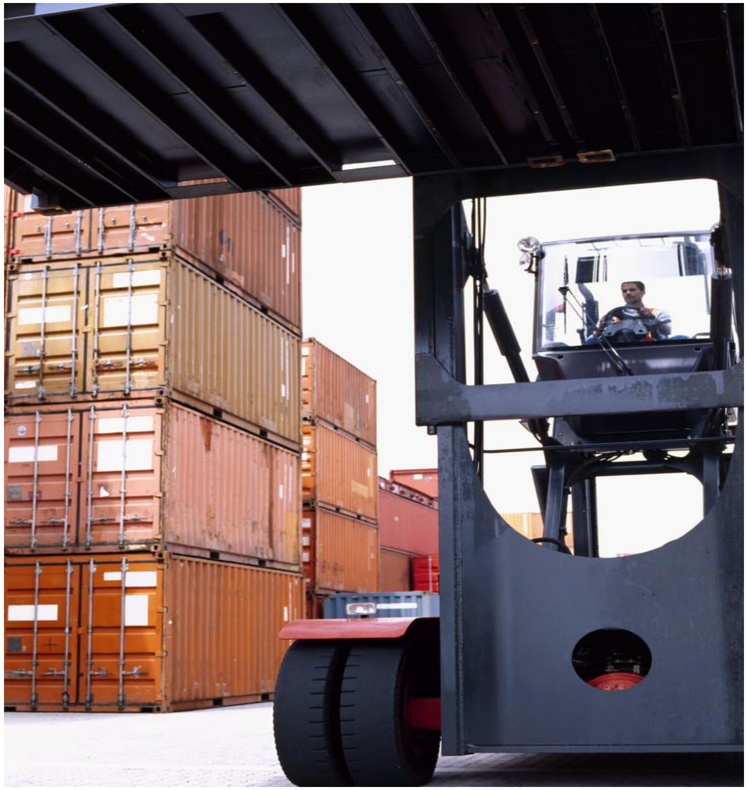
Outline

- Motivation for Port-of-Entry
- Problem Formulation
- Solution
- Conclusion

Motivation

- Large volume of incoming cargo at port-of-entry (POE) to U.S.
- Optimization problem:
  - Minimize risk of contraband material
  - Minimize earliness/tardiness from processing
- Various kinds of risks:
  - Nuclear material
  - Drugs
  - Other hazardous materials

Motivation



- Profile the containers based on:
  - Origin
  - Destination
  - Intelligence
- Two components to consider:
  - Allocation of containers to inspection stations
  - Schedule of inspection of containers at each individual station

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Problem Formulation

- Assumptions:
  - *n* containers, *k* inspection stations
  - Each inspection station can only test for one particular risk
    - *k* risk categories
- A vector to express risk profiles: $x = (x_1, x_2, ..., x_k)$
- Assign $y_{ij} = 1$ if container *i* is inspected at station *j*, and $y_{ij} = 0$ if container *i* is not inspected at station *j*

Young's Model

$$\min\,(1-w_T)\sum_{i=1}^{n}\sum_{j=1}^{k}\pi_{ij}[1-PTR_j y_{ij}]\prod_{s\neq j,\,s=1}^{k}[1-PFR_s y_{is}]+w_T\sum_{i=1}^{n}T_i$$

w_T	weight of tardiness	PTR_j	prob. of true reject at station j
i	container i $(i=1,2,\ldots n)$	$y_{ij}=$	1 if cont. i is inspected by station j; 0 otherwise
j	inspection station j $(j=1,2,..k)$	PFR_j	prob. of false reject at station j
π_{ij}	prob. of risk category j present in cont. i	T_i	tardiness of cont. i

Young's Model

$$\min (1 - w_T) \underbrace{\sum_{i=1}^{n} \sum_{j=1}^{k} \pi_{ij} [1 - PTR_j y_{ij}] \prod_{s \neq j, s=1}^{k} [1 - PFR_s y_{is}]}_{\boxed{\text{Risk}}} + w_T \sum_{i=1}^{n} T_i$$

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π_{ij}	prob. of risk category j present in cont. i	T_i	tardiness of cont. i

Our Model

$$\min\,(1 - w_E - w_T)\sum_{i=1}^{n}\sum_{j=1}^{k}\pi_{ij}[1 - PTR_j y_{ij}]\prod_{s\neq j,\, s=1}^{k}[1 - PFR_s y_{is}] +$$

$$w_E\sum_{i=1}^{n}(D_i - C_i)x_i + w_T\sum_{i=1}^{n}(C_i - D_i)(1 - x_i)$$

Our Model

$$\min\,(1 - w_E - w_T)\underbrace{\sum_{i=1}^{n}\sum_{j=1}^{k}\pi_{ij}[1 - PTR_j y_{ij}]\prod_{s\neq j,\, s=1}^{k}[1 - PFR_s y_{is}]}_{\boxed{\text{Risk}}} +$$

$$w_E\sum_{i=1}^{n}(D_i - C_i)x_i + w_T\sum_{i=1}^{n}(C_i - D_i)(1 - x_i)$$

Our Model

$$\min\,(1 - w_E - w_T)\sum_{i=1}^{n}\sum_{j=1}^{k}\pi_{ij}[1 - PTR_j y_{ij}]\prod_{s\neq j,\, s=1}^{k}[1 - PFR_s y_{is}] +$$

$$w_E \underbrace{\sum_{i=1}^{n}(D_i - C_i)x_i}_{\text{Earliness}} + w_T\sum_{i=1}^{n}(C_i - D_i)(1 - x_i)$$

Our Model

$$\min\,(1 - w_E - w_T)\sum_{i=1}^{n}\sum_{j=1}^{k}\pi_{ij}[1 - PTR_j\, y_{ij}]\prod_{s\neq j,\, s=1}^{k}[1 - PFR_s\, y_{is}] +$$

$$w_E\sum_{i=1}^{n}(D_i - C_i)x_i + w_T\underbrace{\sum_{i=1}^{n}(C_i - D_i)(1 - x_i)}_{\text{Tardiness}}$$

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Allocation

- Given specifications of n, k, PTR, PFR, π_{ij}, w_E, and w_T, we can determine the optimal allocation to reduce risk
- As is to be expected, it is optimal to inspect every container at every station

- $n = 3$, $k = 3$

Stations

Containers	A	B	C
1	1	1	0
2	0	1	1
3	1	1	1

Schedule

- Given an allocation, it is possible to solve for the optimal schedule
- Complete enumeration
- Other heuristics:
  - Earliest Due Date (EDD)
  - Longest Processing Time (LPT)
  - Shortest Processing Time (SPT)

Schedule

- Given the allocation:

Stations

Containers	A	B	C
1	1	1	
2		2	2
3	3	3	3

  - n = 3, k = 3
  - (2!)(3!)(2!) = 24 possible schedules

Schedule

1	1	2
3	2	3
	3	

1	1	2
3	3	3
	2	

1	1	3
3	3	2
	2	

1	2	2
3	1	3
	3	

1	2	3
3	1	2
	3	

1	1	3
3	2	2
	3	

1	2	2
3	2	3
	1	

1	2	3
3	3	2
	1	

1	3	2
3	1	3
	2	

1	3	3
3	1	2
	2	

1	3	2
3	2	3
	1	

1	3	3
3	2	2
	1	

3	1	2
1	2	3
	3	

3	1	3
1	2	2
	3	

3	1	2
1	3	3
	2	

3	1	3
1	3	2
	2	

3	2	3
1	1	2
	3	

3	2	2
1	1	3
	3	

3	2	2
1	3	3
	1	

3	2	3
1	3	2
	1	

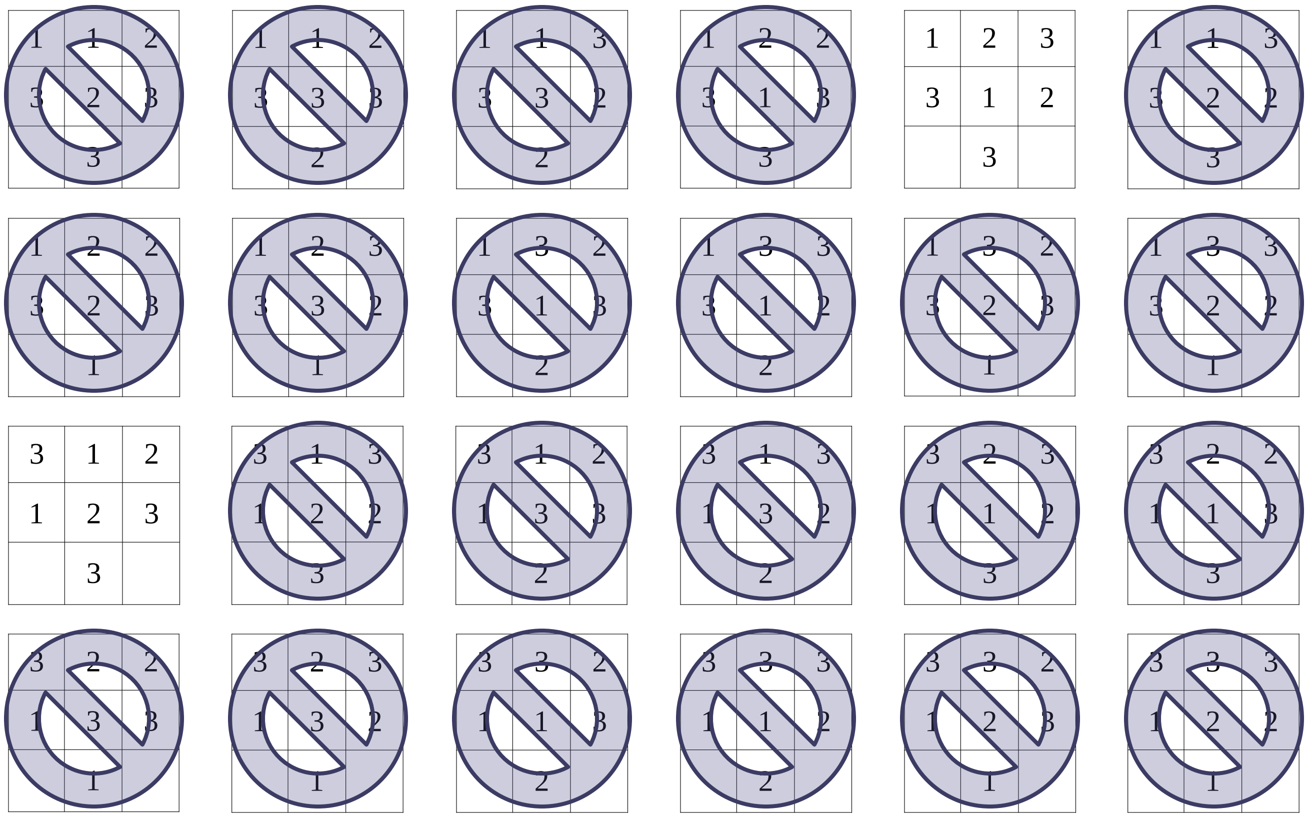
3	3	2
1	1	3
	2	

3	3	3
1	1	2
	2	

3	3	2
1	2	3
	1	

3	3	3
1	2	2
	1	

Schedule



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Center for Discrete Mathematics & Theoretical Computer Science
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Schedule

Schedule A

Stations

Time	A	B	C	
	1	2	3	1h
	3	1	2	2h
		3		3h

Container Due Dates:
1:2h 2:3h 3:5h
Container Completion Time:
1: 2h 2:2h 3:3h
Total earliness: 3h

Schedule B

Stations

Time	A	B	C	
	3	1	2	1h
	1	2	3	2h
		3		3h

Container Due Dates:
1:2h 2:3h 3:5h
Container Completion Time:
1:2h 2:2h 3:3h
Total earliness: 3h

Solving Simultaneously

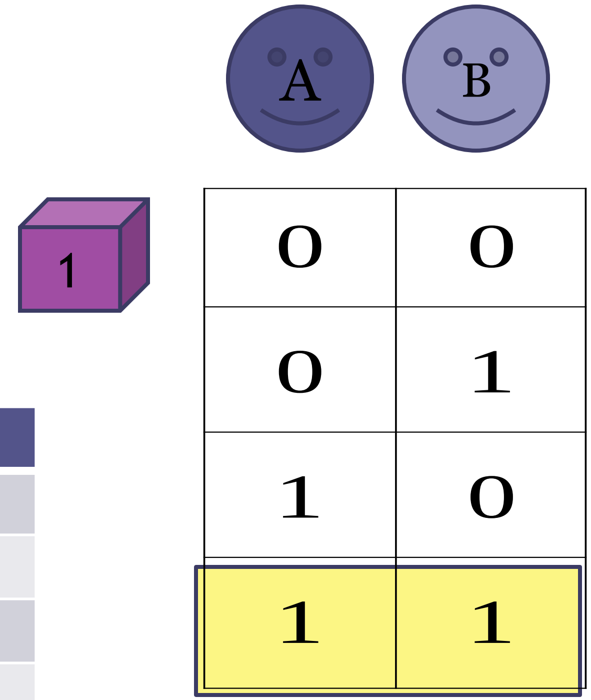
- Comparing combinations
- Each combination has two scores associated:
  - Risk score from allocation
  - Tardiness/earliness from schedule
  - "Normalize" each of these scores to find total cost of the allocation/schedule combination
- The combination with the minimum function value is the ideal

Solving Simultaneously

- Normalization: $$\frac{\text{Calculated} - \text{Minimum}}{\text{Maximum} - \text{Minimum}}$$
- R_{min} = Risk score associated with allocation that inspects at every station
  - T_{max} = Tardiness accumulated from feasible schedule associated with this allocation
- R_{max} = Risk score associated with allocation that inspects at no stations
  - E_{max} = Earliness accumulated from feasible schedule associated with this allocation
- $T_{min} = 0$, $E_{min} = 0$

Example

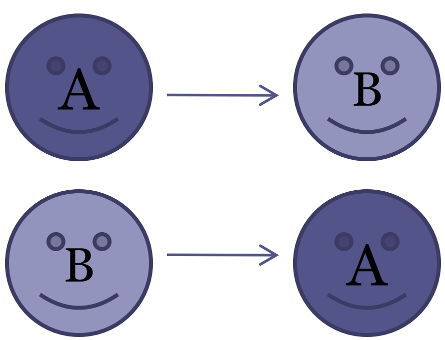
- n = 1, k = 2
- First determine optimal allocation
- Assumptions
  - PTR = .9, .99
  - PFR = .1, .01
  - π_{ij} = .3, .8
  - w_E = .2
  - w_T = .7

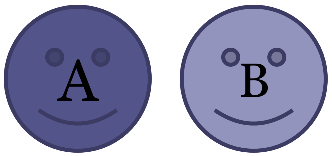


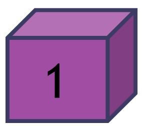
Allocation	Risk Score	Normalized
0 0	.1100	1
0 1	.0305	.25
1 0	.0750	.67
1 1	.0037	0

Example

- n = 1, k = 2
- Allocation
  - To minimize risk, inspect container at all stations
  - Risk score: 0
- Two possible schedules



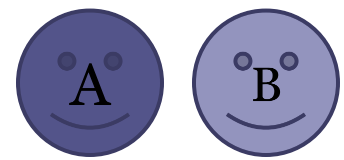


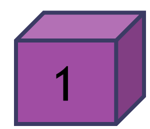


0	0
0	1
1	0
1	1

Example

- Two possible schedules
- A →B
  - Due time: 1h
  - Processing time: 1h
  - Start at A: 0h
  - Start at B: 1h
  - Finish: 2h
  - Tardiness: 1h

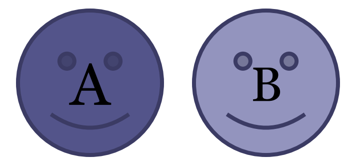


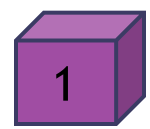


0	0
0	1
1	0
1	1

Example

- Two possible schedules
- B →A
  - Due time: 1h
  - Processing time: 1h
  - Start at B: 0h
  - Start at A: 1h
  - Finish: 2h
  - Tardiness: 1h





0	0
0	1
1	0
1	1

Example

Allocation	Tardiness	Earliness
1 1	1	0

- Normalize Tardiness:

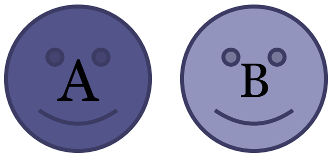
$$\frac{\text{Calculated} - \text{Minimum}}{\text{Maximum} - \text{Minimum}}$$

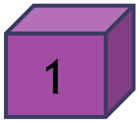
$$\frac{1 - 0}{1 - 0} = 1$$

- Weight of tardiness (w_T): .7
  - Weighted tardiness = (1)(.7) = .7
- Total objective function value = 0 +.7 = .7

Example

- Consider another allocation
  - Risk score: .25
- One schedule:
  - Due time: 1h
  - Start at B: 0h
  - Finish at B: 1h
  - 0 tardiness, 0 earliness
  - Total objective function value: .25
- Optimal combination overall





0	0
0	1
1	0
1	1

Example

- $n = 1$, $k = 2$
- First determine optimal allocation
- Assumptions
  - $PTR = .9, .99$
  - $PFR = .1, .01$
  - $\pi_{ij} = .3, .8$
  - $w_E = .2$
  - $w_T = .7$

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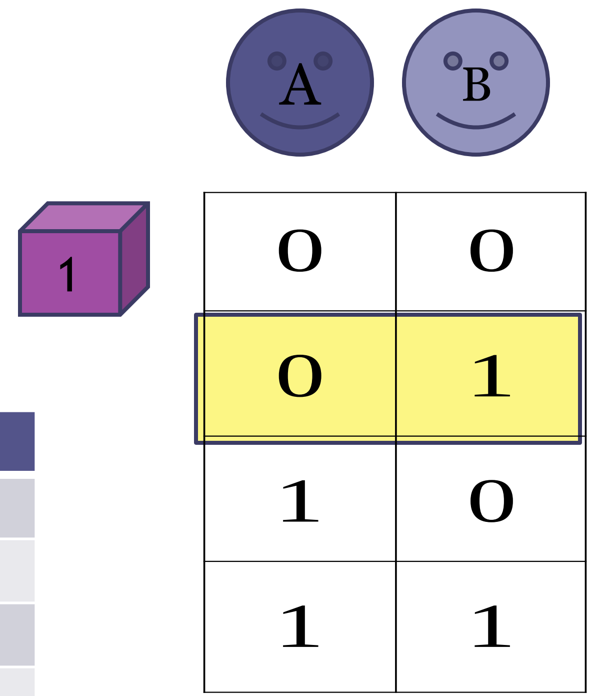
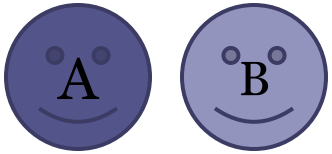
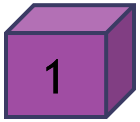


Example

- Consider another allocation
  - Risk score: .25
- One schedule:
  - Due time: 1h
  - Start at B: 0h
  - Finish at B: 1h
  - 0 tardiness, 0 earliness
  - Total function value: .25
- Optimal combination overall





0	0
0	1
1	0
1	1

Example

Allocation	Earliness	Normalized	$w_E = .2$
0 0	1	1	.2
0 1	0	0	0
1 0	0	0	0
1 1	0	0	0

Allocation	Tardiness	Normalized	$w_T = .7$
0 0	0	0	0
0 1	0	0	0
1 0	0	0	0
1 1	1	1	.7

Example

Allocation	Risk	Tardiness	Earliness	Total
0 0	1	0	.2	1.2
0 1	.25	0	0	.25
1 0	.67	0	0	.67
1 1	0	.7	0	.7

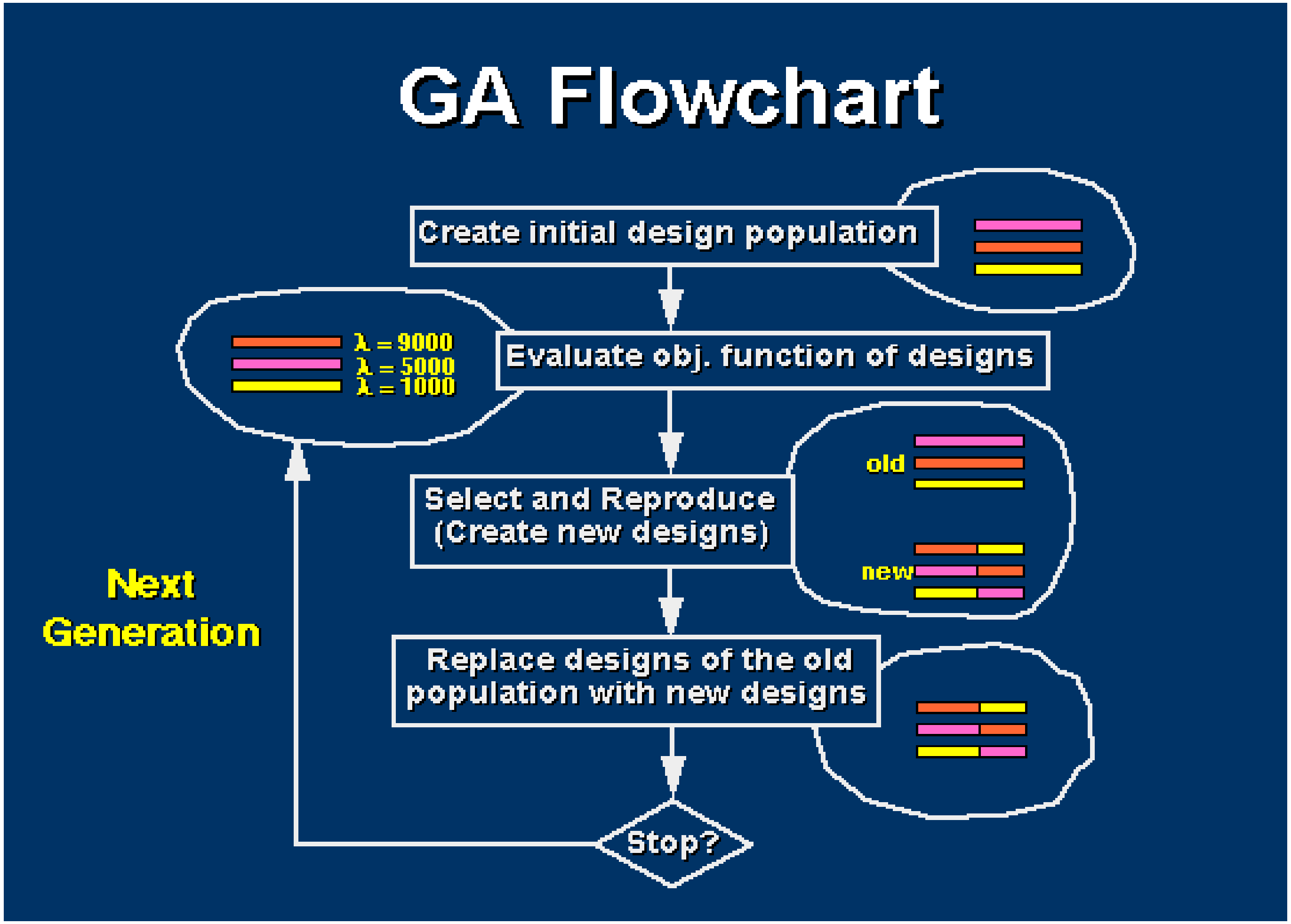
Larger Examples

- Problem gets large very quickly
- For example, n = 3, k = 3
  - 2^9 = 512 possible allocations

Stations

Containers	A	B	C
1	0 or 1	0 or 1	0 or 1
2	0 or 1	0 or 1	0 or 1
3	0 or 1	0 or 1	0 or 1

  - Up to (3!)(3!)(3!) = 216 possible schedules within allocations



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Conclusion

- Problem is NP-hard
- Still no solution to solve allocation and schedule simultaneously
  - Our algorithm does well

Questions?

Thank you!

Ren.clarke@yahoo.com
Darlena.kern@pepperdine.edu

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